$$\alpha_2 = 36\dot{x}_0 y_0 - 30y_0^2 - 10\dot{x}_0^2 - 3x_0\dot{y}_0 - 2\dot{y}_0^2 - 3x_0^2 - z_0^2 - 2\dot{z}_0^2$$

$$\alpha_3 = -3x_0y_0 + 2\dot{x}_0\dot{y}_0 - 6y_0\dot{y}_0 - 2z_0\dot{z}_0$$

$$\alpha_4 = \tfrac{1}{4}\dot{y}_0^2 - \dot{x}_0^2 + 3\dot{x}_0y_0 - \tfrac{9}{4}y_0^2 + \tfrac{1}{4}\dot{z}_0^2 - \tfrac{1}{4}z_0^2$$

$$\alpha_5 = -\tfrac{1}{2}\dot{y}_0(3y_0 - 2\dot{x}_0) + \tfrac{1}{2}z_0\dot{z}_0$$

$$\alpha_6 = -3\dot{y}_0(2y_0 - \dot{x}_0)$$

$$\alpha_7 = -3(2y_0 - \dot{x}_0)(2\dot{x}_0 - 3y_0)$$

$$\beta_0 = 3[(x_0^2/2) + \dot{x}_0^2 + \tfrac{7}{2}y_0^2 - \tfrac{1}{2}\dot{y}_0^2 - 4\dot{x}_0y_0 + \tfrac{1}{4}(z_0^2 + \dot{z}_0^2)]$$

$$\beta_1 = -3(x_0 + 2\dot{y}_0)(2y_0 - \dot{x}_0)$$

$$\beta_2 = -\tfrac{9}{2}(2y_0 - \dot{x}_0)^2$$

$$\beta_3 = 12y_0\,\dot{y}_0 + 6x_0y_0 - 7\dot{x}_0\dot{y}_0 - 3x_0\dot{x}_0 + z_0\dot{z}_0$$

$$\beta_4 = -\tfrac{3}{2}x_0^2 - 5\dot{x}_0^2 - 15y_0^2 + 2\dot{y}_0^2 + 18\dot{x}_0y_0 - \dot{z}_0^2 - \tfrac{1}{2}z_0^2$$

$$\beta_5 = \dot{y}_0(2\dot{x}_0 - 3y_0) - (z_0\dot{z}_0/2)$$

$$\beta_6 = \tfrac{9}{2}y_0^2 + 2\dot{x}_0^2 - \tfrac{1}{2}\dot{y}_0^2 + \tfrac{1}{4}\dot{z}_0^2 - \tfrac{1}{4}z_0^2 - 6\dot{x}_0y_0$$

$$\beta_7 = -3(2\dot{x}_0 - 3y_0)(2y_0 - \dot{x}_0)$$

$$\beta_8 = 3\dot{y}_0(2y_0 - \dot{x}_0)$$

$$\gamma_0 = \tfrac{3}{2}[\dot{y}_0\,\dot{z}_0 + z_0(2\dot{x}_0 - 3y_0)]$$

$$\gamma_1 = 3y_0\dot{z}_0 - \dot{x}_0\dot{z}_0 + z_0\dot{y}_0$$

$$\gamma_2 = -\tfrac{3}{2}\dot{y}_0\dot{z}_0 - 2z_0\dot{x}_0 + 3y_0z_0 - \tfrac{1}{2}y_0\,\dot{z}_0$$

$$\gamma_3 = -\tfrac{1}{2}[\dot{z}_0(2\dot{x}_0 - 3y_0) + z_0\dot{y}_0]$$

$$\gamma_4 = -\tfrac{1}{2}[z_0(2\dot{x}_0 - 3y_0) - \dot{z}_0\dot{y}_0]$$

$$\gamma_5 = 3z_0\,(2y_0 - \dot{x}_0)$$

$$\gamma_6 = -3\dot{z}_0(2y_0 - \dot{x}_0)$$

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Determination of Hypersonic Flow Fields by the Method of Characteristics

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The conservation of mass, momentum, and energy in a hypersonic method-of-characteristics solution is examined and large errors are discovered. The source of the error is traced to an assumption used in the normal method of characteristics computer program and not in the theory itself. A new method of determining the local entropy in a rotational field is discussed and shown to reduce the conservation errors drastically.

In Ref. 1, a check was made of the continuity of mass, momentum, and energy in a hypersonic flow field calculated by the method of characteristics. On the basis of the errors found in this check, the authors concluded that certain errors were inevitable, and that corrections to the computed results must be made.

A similar check has been carried out for one problem of a previously published set of real gas method of characteristics solutions.[2] In contrast to Ref. 1 where the continuity checks were carried out in a constant X plane, the mass, momentum, and energy conservation equations were evaluated along right running characteristics from the shock to the body.

The body used in the problem under discussion is shown in Fig. 1. The nose sphere has a radius of 0.25, and is followed by a tangent frustum, which in turn is faired smoothly by a radius segment into a cylindrical afterbody of radius 1.0. The freestream Mach number used was 35 and an altitude of 300,000 ft. The mass-ratio results given in Fig. 1 typify all the results for momentum and energy also. Two curves are given: one showing the results from a conventional method of characteristics method, and one showing the results using a new method proposed below. It is evident that large errors in continuity can and do occur when using a conventional method.

In order to discover the reason for the large conservation errors, consider the method of calculating a normal field point in a rotational method of characteristics procedure. It is assumed that the independent variables are P and δ, the local pressure and flow direction, so that entropy does not occur explicitly in the compatibility relations. Figure 2 is a sketch of a field showing two known points, A and B, which are to be used in determining a third point, C. By using the compatibility relations in finite difference form, the data given at points A and B determine the pressure and flow direction at C. If $\tan\delta$ is assumed to vary linearly between A and B, a quadratic expression can be derived for determining point D, the intersection of the streamline passing through C with the line joining A and B. Since entropy is a constant along streamlines in equilibrium flows, a linear interpolation between A and B for the entropy at D then determines the entropy at C.

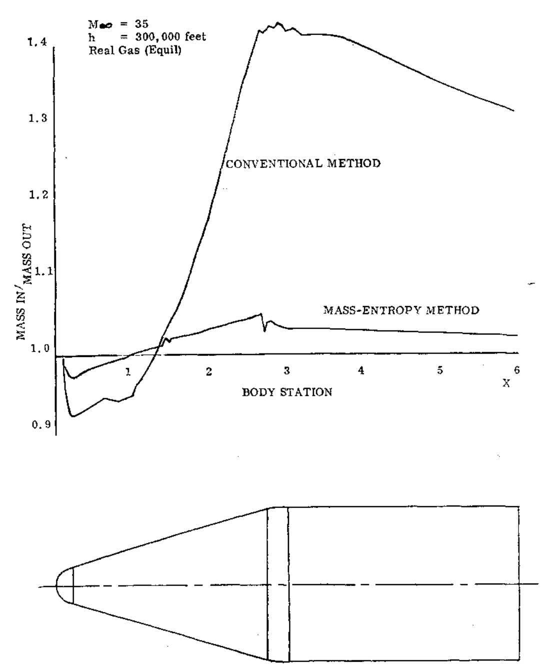

Fig. 1 Continuity check

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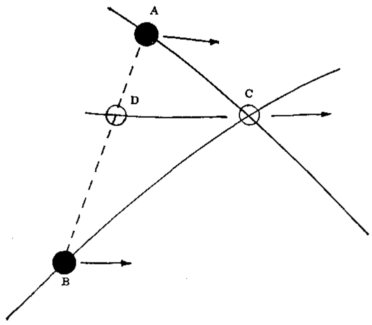

Fig. 2 Field point geometry

The dominant feature of any hypersonic flow field is the appearance of very strong gradients in the flow properties. However, as just noted, the usual numerical method-of-characteristics program relies on an assumption of constant entropy gradient between nearby field points. It is this assumption that introduces the error into the flow field calculations.

The remedy is quite simple. For bodies without secondary shocks, the relation between the value of the mass integral and entropy is established at the shock wave and is invariant (the value of the mass integral assigned to a particular streamline is defined as the amount of mass flowing between the streamline and the body, i.e., a Stokes stream-function). For bodies with secondary shocks, the relation is not invariant, but the philosophy of calculation is the same; the influence of the imbedded shock can be accounted for easily. In order to calculate a new field point properly, one may use this mass-entropy curve in the following manner.

From the data given at points A and B and the compatibility relations in finite difference form, the pressure and flow direction at C are calculated as before. A first guess as to the entropy at C is made by, say, averaging the entropy at A and B. This permits one to determine the Mach number at point C. The mass integral value of the streamline through A is known, and let it be mass_A. The initial value of the mass integral at the shock is, of course,

$$\text{mass}_{\text{shock}} = (2\pi y_{\text{shock}})^{\epsilon}\rho_{\infty}V_{\infty}y_{\text{shock}}$$

The appropriate value of the mass flow at C is then given by

$$\text{mass}_C = \text{mass}_A = (2\pi y)^{\epsilon} <\rho a y> \Delta s$$

where $<\rho a y>$ is the average of the product of density, speed of sound and ordinate at points A and C, $\Delta s = (\Delta x^2 + \Delta y^2)^{1/2}$

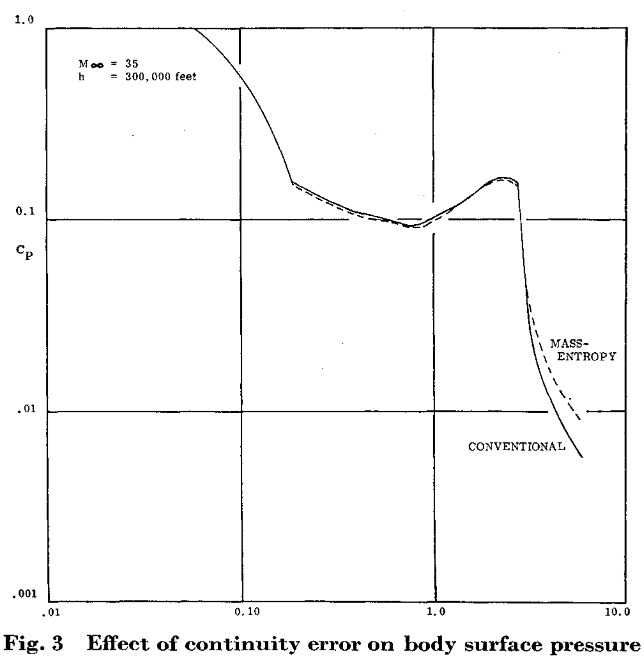

Fig. 3 Effect of continuity error on body surface pressure

and $\epsilon = 0$ or 1 for two-dimensional or axisymmetric flow, respectively. Using the value of mass, and the universal mass-entropy relation established at the shock wave, a new estimate for the entropy can be made. By iterating this process until no further changes occur, a good estimate of the local entropy can be established. The next overall cycle of iteration using the compatibility relations then can be carried out, and the entire process repeated until convergence occurs.

The mass flow ratio using this "mass-entropy" method is given in Fig. 1 where identical mesh sizes were used for both cases. Note that a large reduction in the continuity error has been achieved. It is evident that this new method is superior to the older method of assuming a linear variation of flow properties between adjacent field points.

The influence of the error shown in Fig. 1 on the body pressure distribution is small, as shown in Fig. 3. Only near the base of the body does any appreciable difference occur. The principal damage done by the mass error is in the flow field itself where in some cases an order-of-magnitude difference between quantities calculated by the "mass-entropy" method and the usual practice has been found.

The method of characteristics program used to generate the data presented in this note now has been extended to handle secondary and envelope shocks by the mass-entropy method with the same degree of overall improvement. The mass error value also has proven to be an invaluable guide in uncovering errors in the calculation process.

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Gyroscopic Stabilization of Space Vehicles

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The gyroscopic stabilization of a space vehicle containing a simple disk gyro is investigated. The governing dynamical equations are derived and applied to two specific cases. The first case is a study of the stability of uniform rotation with uniform gyro speed. The second case considers stability of uniform rotation with an oscillating gyro. The results indicate that the stability is highly dependent upon the motion of the gyro and that stabilization may be attained independently of the inertia properties of the vehicle.

Recently there has been an increasing interest in inertia or gyroscopic stabilization. This is especially true in the field of the mechanics of space vehicles where it is advantageous to attain stability independently of external forces or reaction jets. In connection with this, it is the purpose of this paper to investigate such stabilization by seeking stability criteria for a particular space vehicle-gyro system.†

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† The author initially received the idea for this investigation in a course given by T. R. Kane at the University of Pennsylvania.